Cerenkov radiation of spinning particle

I.B. Khriplovich[1]
Budker Institute of Nuclear Physics
630090 Novosibirsk, Russia,
and Novosibirsk University

Abstract

The Cerenkov radiation of a neutral particle with magnetic moment is considered, as well as the spin-dependent contribution to the Cerenkov radiation of a charged spinning particle. The corresponding radiation intensity is obtained for an arbitrary value of spin and for an arbitrary spin orientation with respect to velocity.

PACS numbers: 01.55.+b General physics, 41.60.Bq Cherenkov radiation

1. The problem of Cerenkov radiation of a neutral particle with magnetic moment, moving in a medium with the refraction index n with velocity $v > c/n$, was considered previously in Refs. [1-6]. The magnetic dipole was modeled therein classically, either by a loop with current, or by a pair of magnetic monopole – antimonopole. Thus obtained results are rather model-dependent, and the conclusion made in Ref. [6] is that the situation with the problem of Cerenkov radiation by a magnetic moment is not exactly clear.

In the present work the problem is addressed as follows. A spinning particle, charged or neutral, with magnetic moment is treated as a point-like one, i.e. it is described by a well-localized wave packet. As to the spin s, it has an arbitrary half-integer or integer value, starting with $s = 1/2$. In particular, in the limit $s \gg 1$ we arrive at the classical internal angular momentum and classical magnetic moment. The result obtained below for a neutral particle with magnetic moment differs considerably from all previous ones. As to the spin-dependent contribution to the Cerenkov radiation of a charged particle, I am not aware of any previous results for it.

Certainly, the effects analyzed here are tiny, too small perhaps to be observed experimentally. Hopefully however, their investigation is of some theoretical interest.

2. We start with the electric and magnetic fields created by a point-like neutral particle with magnetic moment $e\,\mathbf{s}\,g/(2m) = (es\,g/(2m))\,\boldsymbol{\sigma}$; here and below g is the g-factor, and $\boldsymbol{\sigma} = \mathbf{s}/s$. Of course, for $s = 1/2$, vector $\boldsymbol{\sigma}$ consists of the common spin σ-matrices, and in the classical limit $s \gg 1$, $\boldsymbol{\sigma}$ is just a unit vector directed along $\mathbf{s}$. In the particle rest frame, the four-dimensional current density is

$$j_\alpha^{(rf)} = (0, \mathbf{j}^{(rf)}) = \frac{es\,g}{2m}\left(0, \boldsymbol{\nabla}\times\boldsymbol{\sigma}^{(rf)}\right)\,\delta(\mathbf{r}^{(rf)})\,. \tag{1}$$

In the laboratory frame, we are working in, this Lorentz-transformed current looks formally as follows:

$$j_\alpha = \left(\gamma v(\mathbf{n}\mathbf{j}^{(rf)}),\ \mathbf{j}^{(rf)} - \mathbf{n}(\mathbf{n}\mathbf{j}^{(rf)}) + \gamma\,\mathbf{n}(\mathbf{n}\mathbf{j}^{(rf)})\right); \quad \gamma = 1/\sqrt{1-v^2}, \quad \mathbf{n} = \mathbf{v}/v$$

[1] khriplovich@inp.nsk.su

(we put throughout $c = 1$). Now, we have to go over in $\mathbf{j}^{(rf)}$ from the rest-frame coordinates $\mathbf{r}^{(rf)}$ to the laboratory ones:

$$\mathbf{r}^{(rf)} = (\gamma(x - vt),\, y,\, z)\,.$$

Under this Lorentz transformation,

$$\delta(\mathbf{r}^{(rf)}) = \delta(\gamma(x - vt))\,\delta(y)\,\delta(z) = \frac{1}{\gamma}\,\delta(x - vt)\,\delta(y)\,\delta(z) = \frac{1}{\gamma}\,\delta(\mathbf{r} - \mathbf{v}t)\,.$$

Besides this overall factor $1/\gamma$, the components of gradient transform obviously as follows:

$$\nabla_x^{(rf)}\,\delta(\mathbf{r} - \mathbf{v}t) = \frac{1}{\gamma}\,\nabla_x\,\delta(\mathbf{r} - \mathbf{v}t), \quad \nabla_{y,\,z}^{(rf)}\,\delta(\mathbf{r} - \mathbf{v}t) = \nabla_{y,\,z}\,\delta(\mathbf{r} - \mathbf{v}t).$$

As to the spin operators $\boldsymbol{\sigma}^{(rf)}$, also entering $\mathbf{j}^{(rf)}$, their transformation law is the same as that for $\mathbf{j}^{(rf)}$ itself:

$$\boldsymbol{\sigma} = (\sigma_x\,,\, \sigma_y\,,\, \sigma_z) = \boldsymbol{\sigma}^{(rf)} - \mathbf{n}(\mathbf{n}\boldsymbol{\sigma}^{(rf)}) + \gamma\,\mathbf{n}(\mathbf{n}\boldsymbol{\sigma}^{(rf)}) = (\gamma\,\sigma_x^{(rf)}\,,\, \sigma_y^{(rf)}\,,\, \sigma_z^{(rf)})\,,$$

or

$$\sigma_x^{(rf)} = \frac{1}{\gamma}\,\sigma_x, \quad \sigma_{y,\,z}^{(rf)} = \sigma_{y,\,z}\,.$$

Thus, in the laboratory frame the four-dimensional current density, created by the magnetic moment $(esg/(2m))\,\boldsymbol{\sigma}$, is [2]

$$j_\alpha^g(\mathbf{r}, t) = \frac{es\,g}{2m}\left((\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla}),\; (1 - v^2)\boldsymbol{\nabla}\times\boldsymbol{\sigma} + \mathbf{v}(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla})\right)\,\delta(\mathbf{r} - \mathbf{v}t). \tag{2}$$

We note that this 4-current density, as well as the initial rest-frame one (1), is orthogonal to the 4-velocity u_α: $u_\alpha j_\alpha = 0$. This is an extra check of the above transformations. Let us note also that the current density (2) can be conveniently rewritten as the sum of two four-currents, each of them being conserved by itself:

$$j_\alpha^{1g}(\mathbf{r}, t) = \frac{es\,g}{2m}\,(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla})\,(1,\; \mathbf{v})\;\delta(\mathbf{r} - \mathbf{v}t)\,, \tag{3}$$

$$j_\alpha^{2g}(\mathbf{r}, t) = \frac{es\,g}{2m}\,(1 - v^2)\,(0,\; \boldsymbol{\nabla}\times\boldsymbol{\sigma})\;\delta(\mathbf{r} - \mathbf{v}t)\,. \tag{4}$$

We are interested in the back-reaction of the field created by the current (2) upon the spin of the particle. This interaction is

$$H_g = \int d\mathbf{r} j_\alpha^g(\mathbf{r} - \mathbf{v}t) A_\alpha(\mathbf{r}) = \frac{esg}{2m}\,\boldsymbol{\sigma}\left[\mathbf{H} - \frac{\gamma}{\gamma + 1}\,\mathbf{v}(\mathbf{v}\mathbf{H}) - \mathbf{v}\times\mathbf{E}\right], \tag{5}$$

where both field strengths, $\mathbf{H}$ and $\mathbf{E}$, are taken at the point of spin location $\mathbf{r} = \mathbf{v}t$. This is the usual interaction of the magnetic moment of a relativistic neutral particle with an external electromagnetic field. In fact, we have omitted in the final expression a term proportional to the total derivative of the vector potential, $d\mathbf{A}/dt = \partial\mathbf{A}/\partial t + (\mathbf{v}\boldsymbol{\nabla})\mathbf{A}$, since a total time derivative in interaction does not result at all in observable effects. Moreover, in the present case the vector potential $\mathbf{A}$, together with the current creating it, depends on the combination $\mathbf{r} - \mathbf{v}t$ only, so that this total derivative vanishes identically.

[2] Here and below $(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla}) = \boldsymbol{\sigma}\cdot[\mathbf{v}\times\boldsymbol{\nabla}] = [\boldsymbol{\sigma}\times\mathbf{v}]\cdot\boldsymbol{\nabla}$, etc.

This line of reasoning is generalized easily for the case of a charged particle. To this end one has to supplement the spin current (2) with the following, also conserved, contribution:

$$j_\alpha^{th}(\mathbf{r},t) = -\frac{es}{m}\frac{\gamma}{\gamma+1}\,(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla})\,(1,\ \mathbf{v})\ \delta(\mathbf{r}-\mathbf{v}t). \tag{6}$$

In its turn, this current generates one more contribution to the spin interaction with electromagnetic field:

$$H_{th} = \int d\mathbf{r} j_\alpha^{th}(\mathbf{r}-\mathbf{v}t)A_\alpha(\mathbf{r}) = \frac{es}{m}\boldsymbol{\sigma}\left[\left(1-\frac{1}{\gamma}\right)\mathbf{H} - \frac{\gamma}{\gamma+1}\,\mathbf{v}(\mathbf{v}\mathbf{H}) - \frac{\gamma}{\gamma+1}\,\mathbf{v}\times\mathbf{E}\right], \tag{7}$$

which describes the well-known Thomas precession. In this expression we have omitted as well, and by the same reasons, a term proportional to the total derivative $d\mathbf{A}/dt = \partial\mathbf{A}/\partial t + (\mathbf{v}\boldsymbol{\nabla})\mathbf{A}$. Finally, from now on, we will work with the following total interaction

$$H = H_g + H_{th} = -\frac{es}{2m}\,\boldsymbol{\sigma}\left[\left(g-2+\frac{2}{\gamma}\right)\mathbf{H} - (g-2)\frac{\gamma}{\gamma+1}\,\mathbf{v}(\mathbf{v}\mathbf{H}) - \left(g-\frac{2\gamma}{\gamma+1}\right)\mathbf{v}\times\mathbf{E}\right], \tag{8}$$

and the total spin current

$$\begin{aligned} j_\alpha(\mathbf{r},t) &= j_\alpha^g(\mathbf{r},t) + j_\alpha^{th}(\mathbf{r},t) \\ &= \frac{es}{2m}\left\{\left(g-\frac{2\gamma}{\gamma+1}\right)(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla})\,(1,\ \mathbf{v})\ + g(1-v^2)\,(0,\ \boldsymbol{\nabla}\times\boldsymbol{\sigma})\right\}\,\delta(\mathbf{r}-\mathbf{v}t). \end{aligned} \tag{9}$$

Hamiltonian (8) not only generates the spin precession, including of course the Thomas effect. It produces as well the relativistic Stern-Gerlach force

$$\mathbf{F} = -\boldsymbol{\nabla}H. \tag{10}$$

Obviously, this force results in the energy loss and therefore is antiparallel to the velocity $\mathbf{v}$ of the spinning particle. Thus, the energy loss per unit time, or the (positive) radiation intensity, is

$$I = -\mathbf{F}\mathbf{v} = (\mathbf{v}\boldsymbol{\nabla})H. \tag{11}$$

Let us note here that the field strengths $\mathbf{H}$, $\mathbf{E}$, being created by the current density $j_\alpha(\mathbf{r},t)$, depend themselves on the non-commuting operators $\boldsymbol{\sigma}$. Therefore, to guarantee that expression (11) is hermitian, one should, strictly speaking, properly symmetrize the products of σ-operators therein. In fact, however, the final result (see (21) below) proves to be hermitian automatically, without extra efforts.

3. The derivation in this section, resulting in general expression (21) (see below) for the spectral intensity, follows essentially that applied in Ref. [7] to the problem of the common Cerenkov radiation.

We will calculate the radiation intensity by going over to the Fourier transforms $\mathbf{H}_\mathbf{k}$ and $\mathbf{E}_\mathbf{k}$ of the field strengths, defined as follows:

$$\mathbf{H}(\mathbf{r}-\mathbf{v}t) = \int d^3k\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{v}t)}\,\mathbf{H}_\mathbf{k}, \quad \mathbf{E}(\mathbf{r}-\mathbf{v}t) = \int d^3k\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{v}t)}\,\mathbf{E}_\mathbf{k}.$$

For our purpose, the wave vectors $\mathbf{k}$ are conveniently decomposed into the components parallel to the velocity $\mathbf{v}$ and orthogonal to it: $\mathbf{k} = \mathbf{q} + \mathbf{n}\,\omega/v$, $\omega = \mathbf{k}\mathbf{v}$, $(\mathbf{q}\mathbf{v}) = 0$. Then, at the position of the point-like source we have

$$(\mathbf{v}\boldsymbol{\nabla})\,\mathbf{H}(\mathbf{r}=\mathbf{v}t) = \int d^3k\, i\omega\,\mathbf{H}_\mathbf{k} = -\frac{1}{v}\int d^2q \int_{-\infty}^{\infty} d\omega\,\omega\,\mathbf{k}\times\mathbf{A}_\mathbf{k}, \tag{12}$$

$$(\mathbf{v}\boldsymbol{\nabla})\,\mathbf{E}(\mathbf{r}=\mathbf{v}t) = \int d^3k\, i\omega\, \mathbf{E}_{\mathbf{k}} = -\frac{1}{v}\int d^2q \int_{-\infty}^{\infty} d\omega\,\omega\,(\omega\mathbf{A}_{\mathbf{k}} - \mathbf{k}\phi_{\mathbf{k}}), \tag{13}$$

where $\phi_{\mathbf{k}}$ and $\mathbf{A}_{\mathbf{k}}$ are the Fourier transforms of the electromagnetic scalar and vector potentials.

In the generalized Lorenz gauge

$$\mathrm{div}\mathbf{A} + \frac{\partial \hat{\varepsilon}\phi}{\partial t} = 0\,,$$

the wave equations for potentials are

$$\hat{\varepsilon}\left(\Delta\phi \,-\, \hat{\varepsilon}\,\frac{\partial^2\phi}{\partial t^2}\right) \,=\, -\,4\pi j_0\,(\mathbf{r}-\mathbf{v}t) \,=\, -\,4\pi\frac{es}{2m}\left(g - \frac{2\gamma}{\gamma+1}\right)(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla})\,\delta(\mathbf{r}-\mathbf{v}t)\,, \tag{14}$$

$$\Delta\mathbf{A} \,-\, \hat{\varepsilon}\,\frac{\partial^2\mathbf{A}}{\partial t^2} \,=\, -\,4\pi\mathbf{j}\,(\mathbf{r}-\mathbf{v}t)$$

$$= -\,4\pi\frac{es}{2m}\left\{\left(g - \frac{2\gamma}{\gamma+1}\right)(\boldsymbol{\sigma}\mathbf{v}\boldsymbol{\nabla})\mathbf{v} \;+ g(1-v^2)\;\boldsymbol{\nabla}\times\boldsymbol{\sigma}\right\}\,\delta(\mathbf{r}-\mathbf{v}t). \tag{15}$$

Here the "dielectric constant" $\hat{\varepsilon}$ should be understood as an operator; we use below its Fourier-transform $\varepsilon(\omega)$. As to the permeability $\mu(\omega)$, for the frequencies of interest to us, it can be put equal to unity.

Now, for the Fourier transforms of the potentials we obtain

$$\phi_{\mathbf{k}} \,=\, \frac{i}{2\pi^2}\,\frac{1}{\varepsilon(\omega)}\,\frac{es}{2m}\left(g - \frac{2\gamma}{\gamma+1}\right)\frac{(\mathbf{v}\mathbf{k}\boldsymbol{\sigma})}{k^2-\varepsilon(\omega)\,\omega^2}$$

$$= \frac{i}{2\pi^2}\,\frac{1}{\varepsilon(\omega)}\,\frac{es}{2m}\left(g - \frac{2\gamma}{\gamma+1}\right)\,\frac{(\mathbf{v}\mathbf{q}\boldsymbol{\sigma})}{q^2-[\varepsilon(\omega)\,-1/v^2]\,\omega^2}\,, \tag{16}$$

$$\mathbf{A}_{\mathbf{k}} = \frac{i}{2\pi^2}\,\frac{es}{2m}\,\frac{g(1-v^2)[\mathbf{k}\times\boldsymbol{\sigma}] + (g-2\gamma/(\gamma+1))\;\mathbf{v}(\mathbf{v}\mathbf{k}\boldsymbol{\sigma})}{k^2-\varepsilon(\omega)\,\omega^2}$$

$$= \frac{i}{2\pi^2}\,\frac{es}{2m}\,\frac{g(1-v^2)[(\mathbf{q}+\mathbf{n}\,\omega/v)\times\boldsymbol{\sigma}] + (g-2\gamma/(\gamma+1))\;\mathbf{v}(\mathbf{v}\mathbf{q}\boldsymbol{\sigma})}{q^2-[\varepsilon(\omega)\,-1/v^2]\,\omega^2}\,. \tag{17}$$

After substituting (16) and (17) into (12) and (13), we note that

$$\int d^2q \to \pi\int dq^2, \quad \int d^2q\, q_m \to 0\,, \quad \int d^2q\, q_m\, q_n = \frac{1}{2}\,\delta_{mn}\,\pi\int dq^2\, q^2\,.$$

We note also that

$$\int_{-\infty}^{\infty}\frac{d\omega\,\omega\,q^2}{q^2-[\varepsilon(\omega)\,-1/v^2]\,\omega^2} = \int_{-\infty}^{\infty} d\omega\,\omega\left\{1 + \frac{[\varepsilon(\omega)\,-1/v^2]\,\omega^2}{q^2-[\varepsilon(\omega)\,-1/v^2]\,\omega^2}\right\}$$

$$= \int_{-\infty}^{\infty}\frac{d\omega\,\omega^3[\varepsilon(\omega)\,-1/v^2]}{q^2-[\varepsilon(\omega)\,-1/v^2]\,\omega^2}\,.$$

Then the integral over q^2 is conveniently combined with all explicit dependence on ω into the following overall factor for the spectral intensity:

$$I(\omega)\,\sim\, f(\omega) \,= -\,i\sum\omega^3\int\frac{dq^2}{q^2-[\varepsilon(\omega)\,-1/v^2]\,\omega^2}\,. \tag{18}$$

The symbol $\sum$ in this expression means that one should sum over the signs of the frequency: both $\omega = +|\,\omega|$ and $\omega = -|\,\omega|$ contribute to the intensity $I(\omega)$. All other dependence of the total result on ω is via $\varepsilon(\omega)$ only; in our problem of the Cerenkov radiation, we restrict to the frequencies corresponding to the region of transparency, i.e. to real $\varepsilon(\omega)$ which is an even function of ω.

Let us analyze now expression

$$f(\omega) = -\,i \sum \omega^3 \int \frac{dq^2}{q^2 - [\varepsilon(\omega)\, - 1/v^2]\,\omega^2}$$

entering result (18). The poles of its integrand correspond obviously to the vanishing 4-momentum squared of a photon in the medium. Here, however, one should retain in $\varepsilon(\omega)$ its small imaginary part: $\mathrm{Im}\,\varepsilon(\omega) > 0$ for $\omega > 0$, and $\mathrm{Im}\,\varepsilon(\omega) < 0$ for $\omega < 0$. In other words, the poles of the integrand in $f(\omega)$ tend to the real axis from above for $\omega > 0$, and from below for $\omega < 0$. Therefore, their contributions to the integral are $i\pi$ and $-i\pi$, respectively. As to the real part of the integral, it is an even function of ω (together with $\mathrm{Re}\,\varepsilon(\omega)$), and therefore its contributions to the sum $f(\omega)$ cancel. Coming back to the poles, their contributions to $f(\omega)$ are $i\pi\omega^3$ and $-i\pi(-\omega)^3 = i\pi\omega^3$, where from now on ω is positive. Thus,

$$f(\omega) = 2\pi\,\omega^3,$$

Then quite straightforward (though rather tedious) transformations result in the following expressions for $(\mathbf{v}\boldsymbol{\nabla})\,\mathbf{H}$ and $(\mathbf{v}\boldsymbol{\nabla})\,\mathbf{E}$:

$$\begin{aligned}(\mathbf{v}\boldsymbol{\nabla})\,\mathbf{H}(\mathbf{r}=\mathbf{v}t) \;&=\; \frac{es}{2m}\,\frac{\omega^3\,d\omega}{2v}\left\{-\boldsymbol{\sigma}_\perp\left[\left(g-2+\frac{2}{\gamma}\right)\left(\varepsilon-\frac{1}{v^2}\right)+\frac{2g}{\gamma^2 v^2}\right]\right.\\ &\qquad\left. -\boldsymbol{\sigma}_\parallel\frac{g}{\gamma^2}\left(\varepsilon-\frac{1}{v^2}\right)\right\}, \qquad (19)\\ (\mathbf{v}\boldsymbol{\nabla})\,\mathbf{E}(\mathbf{r}=\mathbf{v}t) \;&=\; \frac{es}{2m}\,\frac{\omega^3\,d\omega}{2v}\left[\left(g-\frac{2\gamma}{\gamma+1}\right)\frac{1}{\varepsilon}\left(\varepsilon-\frac{1}{v^2}\right)+2g(1-v^2)\frac{1}{v^2}\right][\mathbf{v}\times\boldsymbol{\sigma}_\perp]\,; \qquad (20)\end{aligned}$$

here and below, $\boldsymbol{\sigma}_\perp$ and $\boldsymbol{\sigma}_\parallel$ are the components of vector $\boldsymbol{\sigma}$, orthogonal and parallel, respectively, to the velocity $\mathbf{v}$.

Now, plugging these expressions into (8) and (10), we arrive at the final general result for the spectral intensity of Cherenkov radiation by a spinning particle:

$$\begin{aligned}I(\omega)d\omega = \left(\frac{es}{2m}\right)^2\frac{\omega^3\,d\omega}{2v}&\left\{\left[\left(g-2+\frac{2}{\gamma}\right)^2\left(n^2(\omega)-\frac{1}{v^2}\right)-\left(g-2+\frac{2}{\gamma+1}\right)^2\left(v^2-\frac{1}{n^2(\omega)}\right)\right.\right.\\ &\left.\left.+\frac{2g^2}{\gamma^4 v^2}\right]\boldsymbol{\sigma}_\perp^2+\frac{g^2}{\gamma^3}\left(n^2(\omega)-\frac{1}{v^2}\right)\boldsymbol{\sigma}_\parallel^2\right\} \qquad (21)\end{aligned}$$

Few remarks on this result.

One should not bother about its formal singularity in v: anyway, Cerenkov radiation takes place for $v \geq 1/n$ only.

Then, as distinct from the common Cerenkov radiation, here the contribution to the energy loss due to $\boldsymbol{\sigma}_\perp$ does not vanish at the threshold, at $v = 1/n$.

At last, it is not exactly clear at first glance whether the structure

$$\left(g-2+\frac{2}{\gamma}\right)^2\left(n^2-\frac{1}{v^2}\right)-\left(g-2+\frac{2}{\gamma+1}\right)^2\left(v^2-\frac{1}{n^2}\right)+\frac{2g^2}{\gamma^4 v^2} \qquad (22)$$

at $\boldsymbol{\sigma}_\perp^2$ is positively definite (as it should be for arbitrary g and γ!). To prove that this is the case indeed, we note that the discussed quadratic function of g is certainly positively definite at $g \to \infty$ for $v \geq 1/n$. On the other hand, the discriminant d of this quadratic form is negatively definite:

$$d = -4\varepsilon \frac{v^2}{\gamma^2}\left(1 - \frac{1}{n^2 v^2}\right)^2 .$$

So, quadratic form (22) is positively definite indeed.

Of course, in the case of a charged spinning particle the common Cerenkov radiation takes place as well (and is strongly dominating quantitatively). But don't we have then some combined effect, a Cerenkov-type radiation of first order in spin? It is practically obvious, by symmetry reasons, that such an effect should not exist. But let us present somewhat more quantitative arguments. The effect could arise due to the Lorentz force $\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{H})$, with $\mathbf{E}$ and $\mathbf{H}$ generated by spin current density (9). However, the magnetic contribution $e\mathbf{v} \times \mathbf{H}$ to the energy loss $-\mathbf{vF}$ vanishes trivially. As to the corresponding electric contribution $-e\mathbf{vE}(\mathbf{r} = \mathbf{v}t)$ to the energy loss, one can demonstrate explicitly with formulae (16), (17) that it vanishes as well. As explicitly one can demonstrate that the contribution to the energy loss due to the Stern-Gerlach force (11), but now with $\mathbf{H}$ and $\mathbf{E}$ generated by the common convection current $j_\mu(\mathbf{r}, t) = e(1, \mathbf{v})\,\delta(\mathbf{r} - \mathbf{v}t)$, vanishes as well.

4. In conclusion, let us consider some particular cases of general result (21).

Let us start with a neutral particle with a finite magnetic moment μ. For $e \to 0$, $g \to \infty$, and $\mu = esg/(2m) \to$ const, we obtain

$$I(\omega)\,d\omega \;=\; \frac{\mu^2\,\omega^3}{2v}\,d\omega\,\left[\left(n^2 - \frac{1}{v^2} - v^2 + \frac{1}{n^2} + \frac{2}{\gamma^4 v^2}\right)\boldsymbol{\sigma}_\perp^2 + \frac{1}{\gamma^3}\left(n^2 - \frac{1}{v^2}\right)\boldsymbol{\sigma}_\parallel^2\right]. \tag{23}$$

For $s = 1/2$ (e.g. for the Dirac neutrino with a mass and magnetic moment), $\boldsymbol{\sigma}_\perp^2 = \boldsymbol{\sigma}^2 - \sigma_z^2 = 2$ and $(\boldsymbol{\sigma}\mathbf{n})^2 = \sigma_z^2 = 1$. So, here we obtain from (23)

$$I(\omega)\,d\omega \;=\frac{\mu^2\,\omega^3}{v}\,d\omega\;\left[\left(n^2 - \frac{1}{v^2} - v^2 + \frac{1}{n^2}\right) + \frac{1}{2\gamma^3}\left(n^2 - \frac{1}{v^2}\right) + \frac{2}{\gamma^4 v^2}\right]. \tag{24}$$

In the classical limit, $s \gg 1$, radiation intensity (23) goes over into

$$I(\omega)\,d\omega \;=\; \frac{\mu^2\,\omega^3}{2v}\,d\omega\left[\left(n^2 - \frac{1}{v^2} - v^2 + \frac{1}{n^2} + \frac{2}{\gamma^4 v^2}\right)\sin^2\theta + \frac{1}{\gamma^3}\left(n^2 - \frac{1}{v^2}\right)\cos^2\theta\right], \tag{25}$$

where θ is the angle between the spin and velocity.

The opposite limiting case is that of a charged particle with the vanishing g-factor. The effect here is finite and looks as follows:

$$I(\omega)d\omega = \left(\frac{es}{2m}\right)^2 \frac{2\omega^3\,d\omega}{v}\left[\left(\frac{\gamma-1}{\gamma}\right)^2\left(n^2 - \frac{1}{v^2}\right) - \left(\frac{\gamma}{\gamma+1}\right)^2\left(v^2 - \frac{1}{n^2}\right)\right]\boldsymbol{\sigma}_\perp^2. \tag{26}$$

And at last let us mention the case $g = 2$ (for instance, that of electron if one neglects its small anomalous magnetic moment). Here

$$I(\omega)d\omega = \left(\frac{es}{2m}\right)^2 \frac{2\omega^3\,d\omega}{v}\left\{\left[\frac{1}{\gamma^2}\left(n^2 - \frac{1}{v^2}\right) - \frac{1}{(\gamma+1)^2}\left(v^2 - \frac{1}{n^2}\right) + \frac{2}{\gamma^4 v^2}\right]\boldsymbol{\sigma}_\perp^2 \right.$$
$$\left. + \frac{1}{\gamma^3}\left(n^2 - \frac{1}{v^2}\right)\boldsymbol{\sigma}_\parallel^2\right\}. \tag{27}$$

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I am grateful to A.A. Pomeransky for the interest to the work and extremely useful discussions. The work was supported in part by the Russian Foundation for Basic Research through Grant No. 08-02-00960-a.

References

[1] I.M. Frank, Proceedings of the Academy of Sciences of the USSR, Physics, **6**, 3, 1942 (in Russian).

[2] I.M. Frank, in the book "To the Memory of S.I. Vavilov", Nauka, Moscow, 1952, p. 172 (in Russian).

[3] V.L. Ginzburg, in the book "To the Memory of S.I. Vavilov", Nauka, Moscow, 1952, p. 193 (in Russian).

[4] V.L. Ginzburg, Proceedings of Institutions of Higher Education, Radiophysics, **27**, 852, 1984 (in Russian).

[5] V.L. Ginzburg, V.N. Tsytovich, "Transitional Radiation and Transitional Scattering", Nauka, Moscow, 1984 (in Russian).

[6] I.M. Frank, "Vavilov – Cerenkov Radiation. Theoretical Aspects", Nauka, Moscow, 1988 (in Russian).

[7] L.D. Landau, E.M. Lifshitz, "Electrodynamics of Continuous Media", Pergamon Press, Oxford, 1989.